



1.

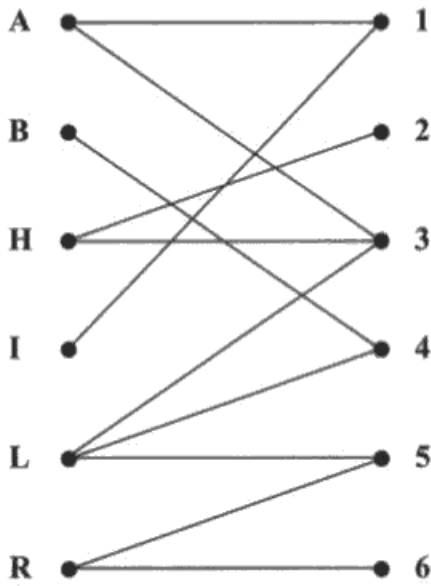


Figure 1

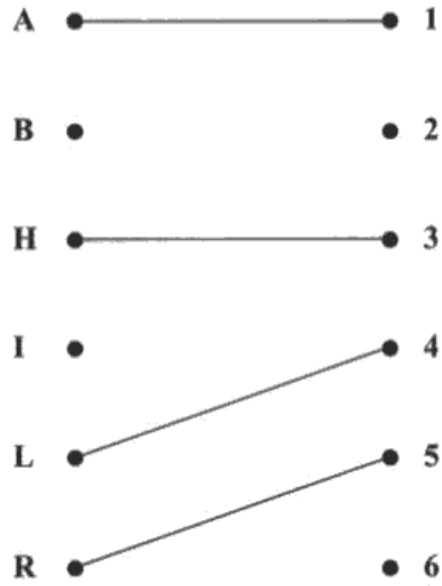


Figure 2

1. a) bipartite graph.

b) Path 1:

$$6 - R = 5 - L = 4 - B$$

change status

$$6 = R - 5 = L - 4 = B$$

improved matching

$$A = 1$$

$$B = 4$$

$$H = 3$$

$$I = \text{unmatched}$$

$$L = 5$$

$$R = 6$$



1 Q01aB

1 Q01bM1

1 Q01bA1

1 Q01bA2

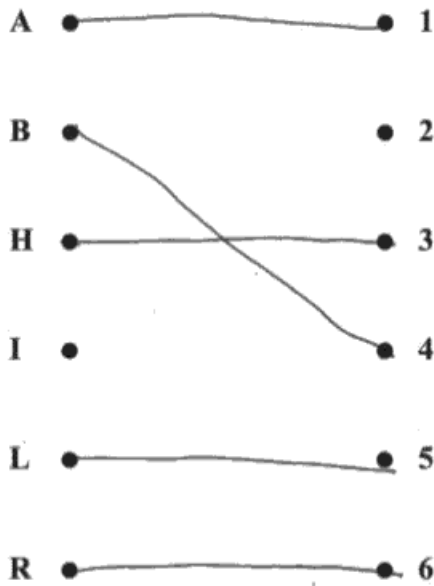
1 Q01bM2

1 Q01bA3

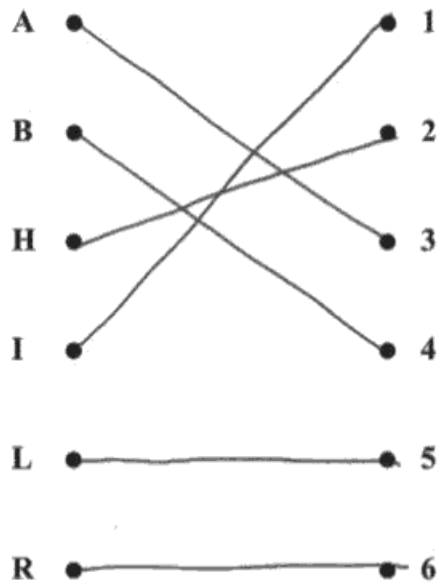
1 Q01bA4



Question 1 continued



initial matching



complete matching



using the improved matching from path 1 as my initial matching for part 2

Path 2 :

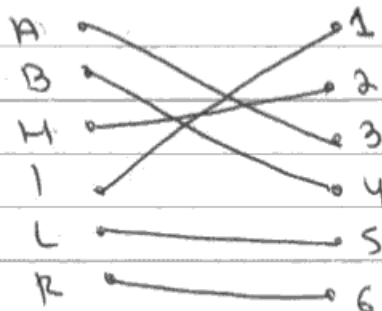
~~2 - H = 3 - L = 5 - R = 6~~

2 - H = 3 - A = 1 - 1

change status

2 = H - 3 = A - 1 = 1

complete matching



$A = 3$   
 $B = 4$   
 $H = 2$   
 $I = 1$   
 $L = 5$   
 $R = 6$

Q1

7

(Total 7 marks)



2.

- a) bin 1: 0.6 0.2 0.4 0.5 0.1
- bin 2: 1.5 0.3
- bin 3: 1.6
- bin 4: 0.7 0.9

PIVOTS

b) 0.6 1.5 1.6 0.7 0.4 0.5 0.7 0.1 0.9 0.3 0.5

0.6 1.5 1.6 0.7 0.9 0.5 0.2 0.4 0.1 0.3 1.6, 0.1

~~1.6 0.6 0.7 0.9 0.5 0.2 0.4 0.3~~

1.6 0.6 1.5 0.7 0.9 0.5 0.2 0.4 0.3 0.1 0.7, 0.4

1.6 1.5 0.9 0.7 0.6 0.5 0.4 0.2 0.3 0.1 0.9, 0.6, 0.3

1.6 1.5 0.9 0.7 0.6 0.5 0.4 0.3 0.2 0.1 1.5, 0.6, 0.2

1.6 1.5 0.9 0.7 0.6 0.5 0.4 0.3 0.2 0.1

All items have been chosen as a pivot and the list is now in descending order. Sort complete.

- c) bin 1 1.6 0.4
- bin 2 1.5 0.5
- bin 3 0.9 0.7 0.3 0.1
- bin 4 0.6 0.2

d) lower bound =  $\frac{6.8}{2} = 3.4$  so 4 bins

Yes uses minimum number of bins because lower bound is 4 bins and part c) also uses 4 bins.

1 Q02aM  
1 Q02aA1  
1 Q02aA2  
1 Q02bM  
1 Q02bA1  
1 Q02bA2  
0 Q02bA3  
1 Q02cM  
1 Q02cA1  
1 Q02cA2  
1 Q02dB





3. (a)

	1	6	2	3	4	5
A	-	15	6	9	-	-
B	15	-	12	-	14	-
C	6	12	-	7	10	-
D	9	-	7	-	11	17
E	-	14	10	11	-	5
F	-	-	-	17	5	-

$AC = 6$

$CD = 7$

$CE = 10$

$EF = 5$

$CB = 12$

total length = 40 days.

Order of arcs: AC, CD, CE, EF, CB

(b)

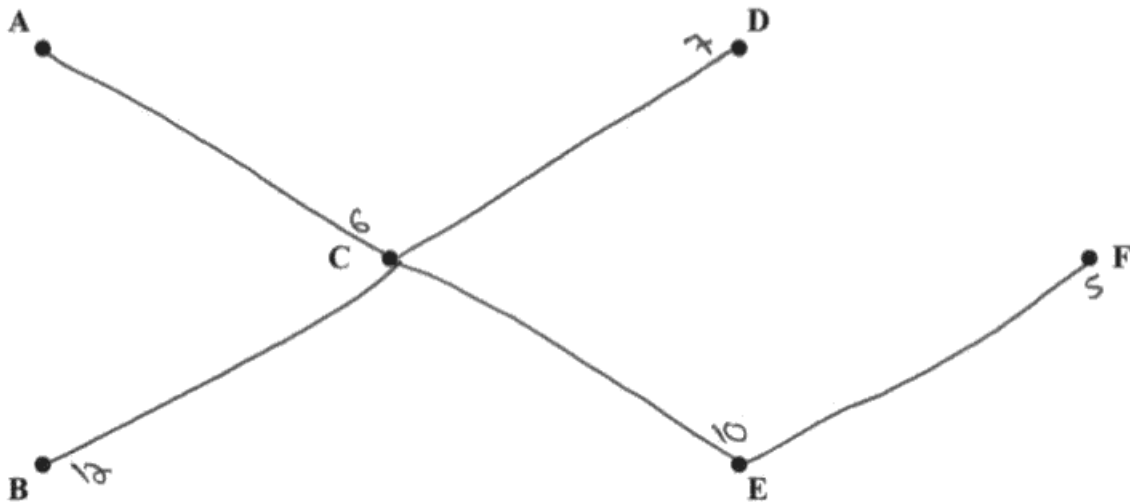


Diagram 1

- 1 Q03aM
- 1 Q03aA1
- 1 Q03aA2
- 1 Q03bB
- 1 Q03cB1
- 1 Q03cB2
- 1 Q03dM
- 1 Q03dA1
- 1 Q03dA2
- 1 Q03eB



Question 3 continued

(c)

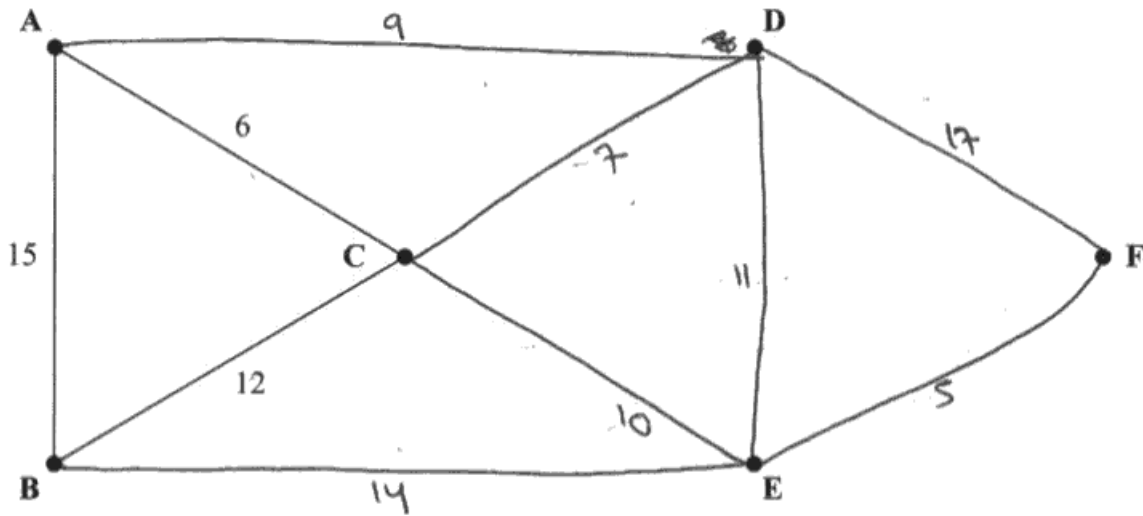


Diagram 2

(d) Arcs listed in ascending order:

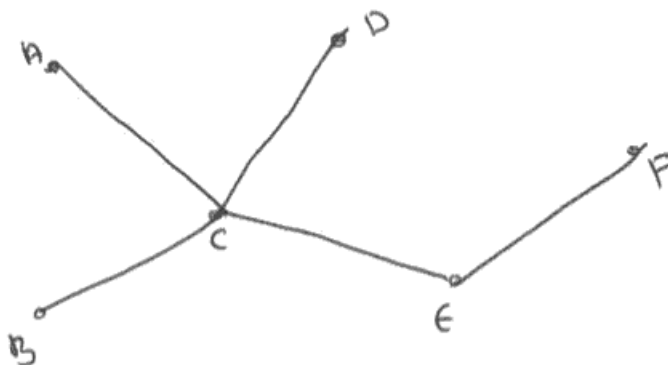
- |                |                                   |
|----------------|-----------------------------------|
| EF = 5 ✓       | BC = 12 ✓                         |
| AC = 6 ✓       | <del>BE = 14</del> BE = 14 reject |
| CD = 7 ✓       | <del>AD = 9</del> AD = 9 reject   |
| AD = 9 reject  | <del>DE = 17</del> DE = 17 reject |
| CE = 10 ✓      |                                   |
| DE = 11 reject |                                   |

(e) Minimum time needed: 40 days

(Total 10 marks)

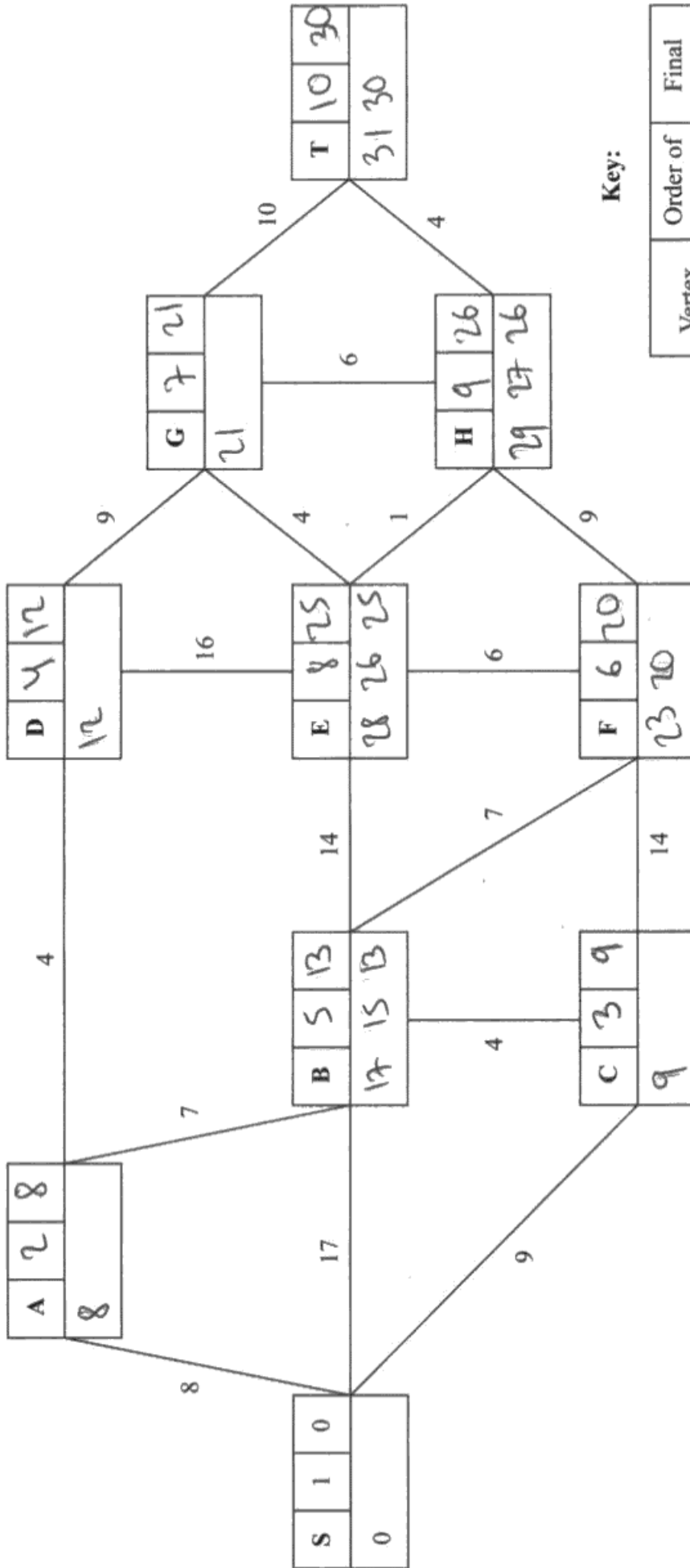
Q3

10



- 1 Q04aM
- 1 Q04aA1
- 0 Q04aA2
- 1 Q04aA3
- 1 Q04aA4
- 1 Q04aA5
- 1 Q04bB1
- 1 Q04bB2

4. (a)



Key:

Vertex	Order of labelling	Final value
Working values (in order)		





**Question 4 continued**

Shortest path from S to T: SADG EHT

Length of shortest path from S to T: 30 miles

(b)

Shortest path from S to T via F: SCBFEHT

Length of shortest path from S to T via F: 31 miles

(Total 8 marks)

Q4

7



- 1 Q05aM
- 1 Q05aA1
- 1 Q05aA2
- 1 Q05aA3
- 1 Q05aA4
- 1 Q05bB1
- 1 Q05bB2
- 1 Q05cM
- 0 Q05cA1
- 1 Q05cA2

5.  $ADEGDFGH EBCBACEDCA$   
 ↑  
 correct

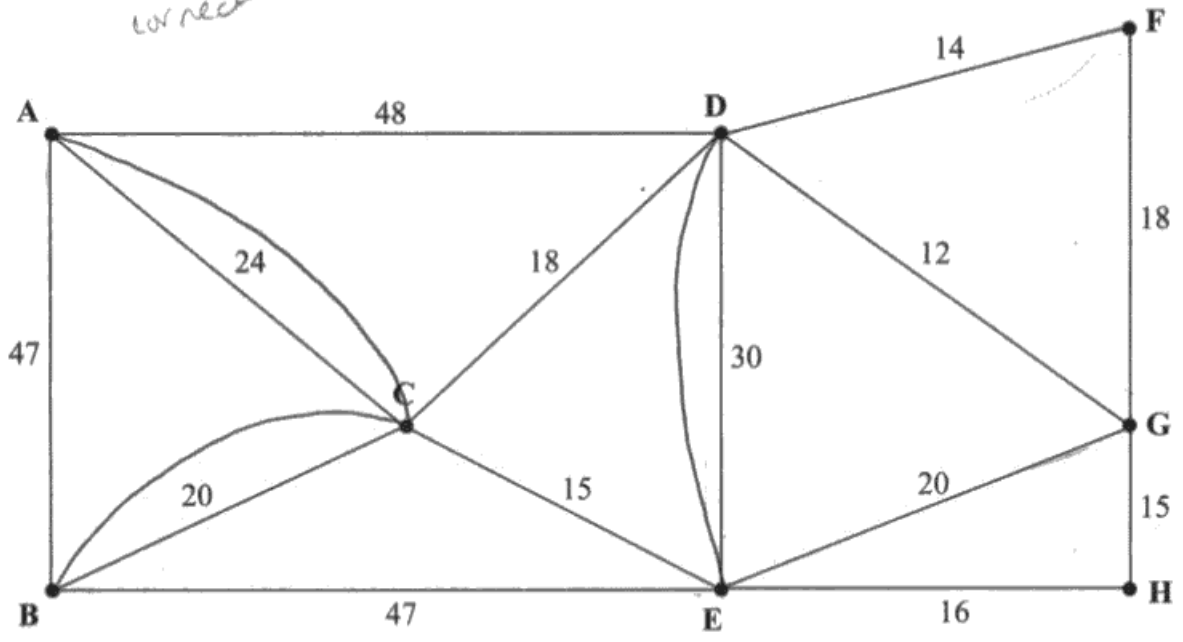


Figure 4

[The total weight of the network is 344 miles]

a) odd nodes: A, B, D, E

$AB + DE = 44 + 30 = 74$  ← shortest so repeat

$AD + BE = 42 + 35 = 77$

$BD + AE = 38 + 39 = 77$

AB + DE is shortest so repeat. Arcs AC CB and DE will need to be repeated.





Question 5 continued

b) total length =  $344 + 74$   
 $= 418$  miles

Possible route: A D E G D F G H E B C B A C  
 $E \rightarrow D \rightarrow C \rightarrow A$

c) E is now even  
 A, B and D are the only odd nodes

AB = 44

AD = 42

BD = 38 ← shortest

BD is shortest so repeat BC CD

Finish at A

Total length =  $344 + 38$   
 $= 382$  miles.

(Total 10 marks)

Q5

9



6.  $x = 2$  seater boats  
 $y = 4$  seater boats

a) total number of boats must be at least 90.  
 Number of 2 seater and 4 seater boats must be equal to or greater than 90.

b)  $2x \leq 3y$   
 $x \leq \frac{3y}{2}$

number of 2 seater boats must be less than or equal to  $\frac{3}{2}$  x the number of 4 seater boats

There must be at least 1.5 times less 2 seater boats than 4 seater boats.

c)  $y \leq x + 30$        $x + y \geq 90$        $2x \leq 3y$

$y = x + 30$	$x + y = 90$	$2x = 3y$
$y = 100$ $x = 70$	$x = 10$ $y = 80$	$x = 15$ $y = 10$
$y = 50$ $x = 20$	$x = 50$ $y = 40$	$x = 30$ $y = 20$
$y = 30$ $x = 0$		$x = 60$ $y = 40$

d) (minimise)  $C = 100x + 300y$ .

- 1 Q06aB
- 1 Q06bB1
- 1 Q06bB2
- 1 Q06cB1
- 1 Q06cB2
- 1 Q06cB3
- 0 Q06cB4
- 1 Q06dB
- 1 Q06eM
- 1 Q06eA
- 0 Q06eB1
- 0 Q06eB2



Question 6 continued

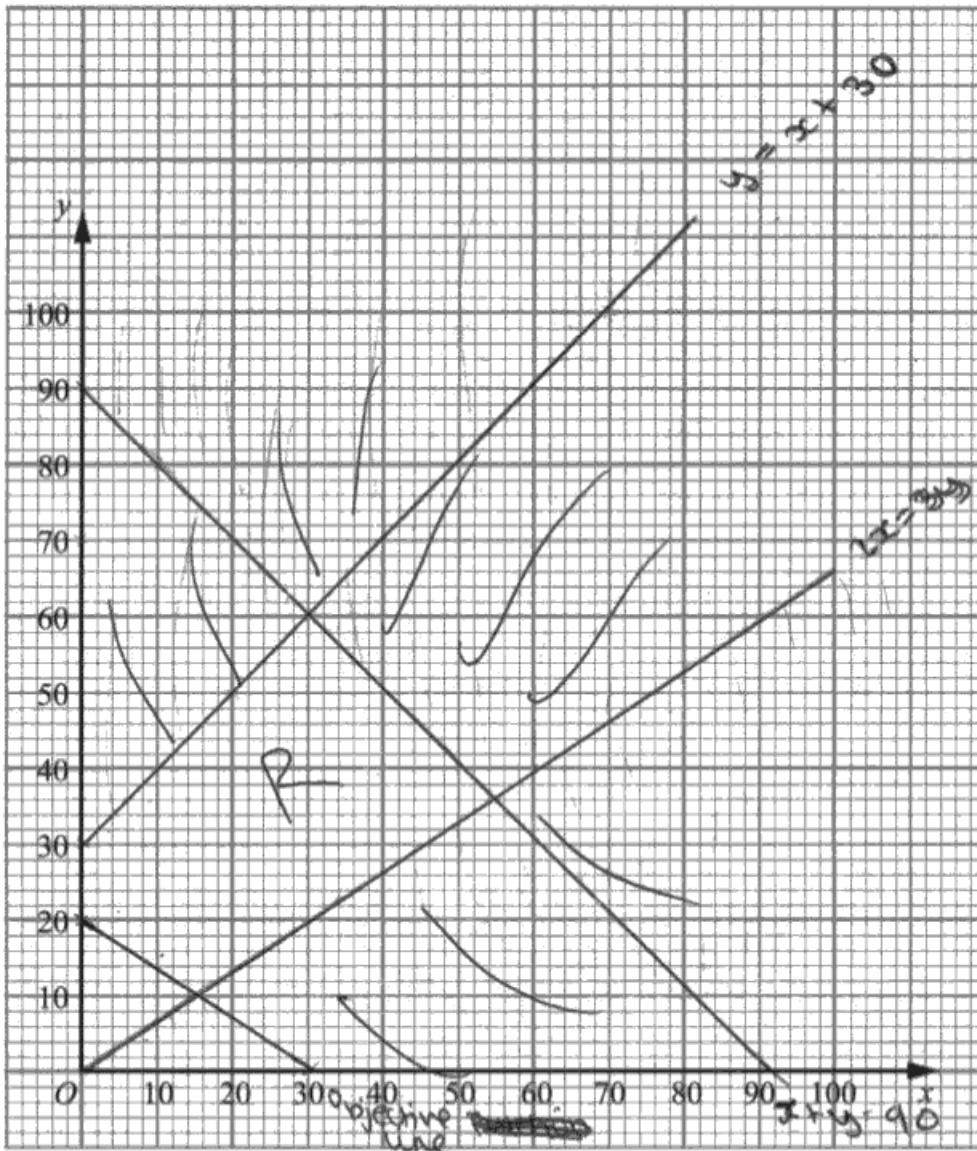


Diagram 1

e)  $C = 100x + 300y$   ~~$C = 100x + 300y$~~

$(0, 20)$	$(0, 30)$	$(54, 36)$	$(30, 60)$
$C = 6000$	$C = 9000$	$C = 16200$	$C = 21000$

Buy 0 2-seater boats and 20 4-seater boats

Cost = £6000

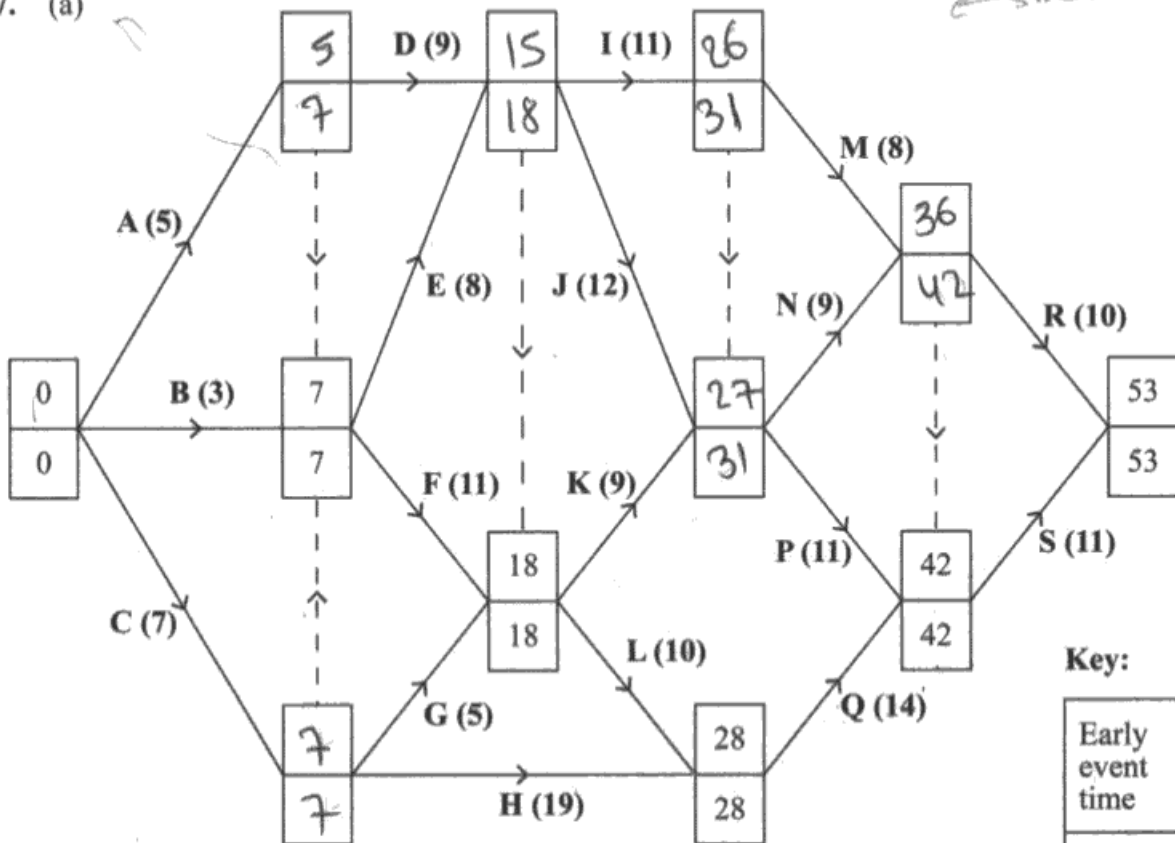
(Total 12 marks)

Q6

9



7. (a)



Key:

Early event time

Late event time

(b)  $42 - (8 + 31) = 3$

(c) i) Project will be ~~delayed~~ <sup>unaffected</sup> by ~~2~~ days.

~~$42 - (11 + 31) = 0$~~

~~$42 - (27 + 11) = 4$~~

$4 - 2 = 2$  2 extra days still left.

(c) ii) Project will be delayed by 2 days

$42 - (14 + 28) = 0$

$0 - 2$

$= -2$

d) lower bound =  $\frac{172}{53} = 3.245$

so 4 workers needed.

Leave blank

- 1 Q07aM1
- 1 Q07aA1
- 1 Q07aM2
- 1 Q07aA2
- 0 Q07bM
- 0 Q07bA
- 1 Q07ciB
- 1 Q7ciiB
- 1 Q07dB
- 1 Q07eM1
- 1 Q07eA1
- 0 Q07eM2
- 0 Q07eA2
- 1 Q07fM
- 1 Q07fA
- 1 Q07gM
- 0 Q07gA



Question 7 continued

~~A B C D E F G H I J K L M N O~~  
~~P Q R S~~

(e)

■ represents the total float on each activity

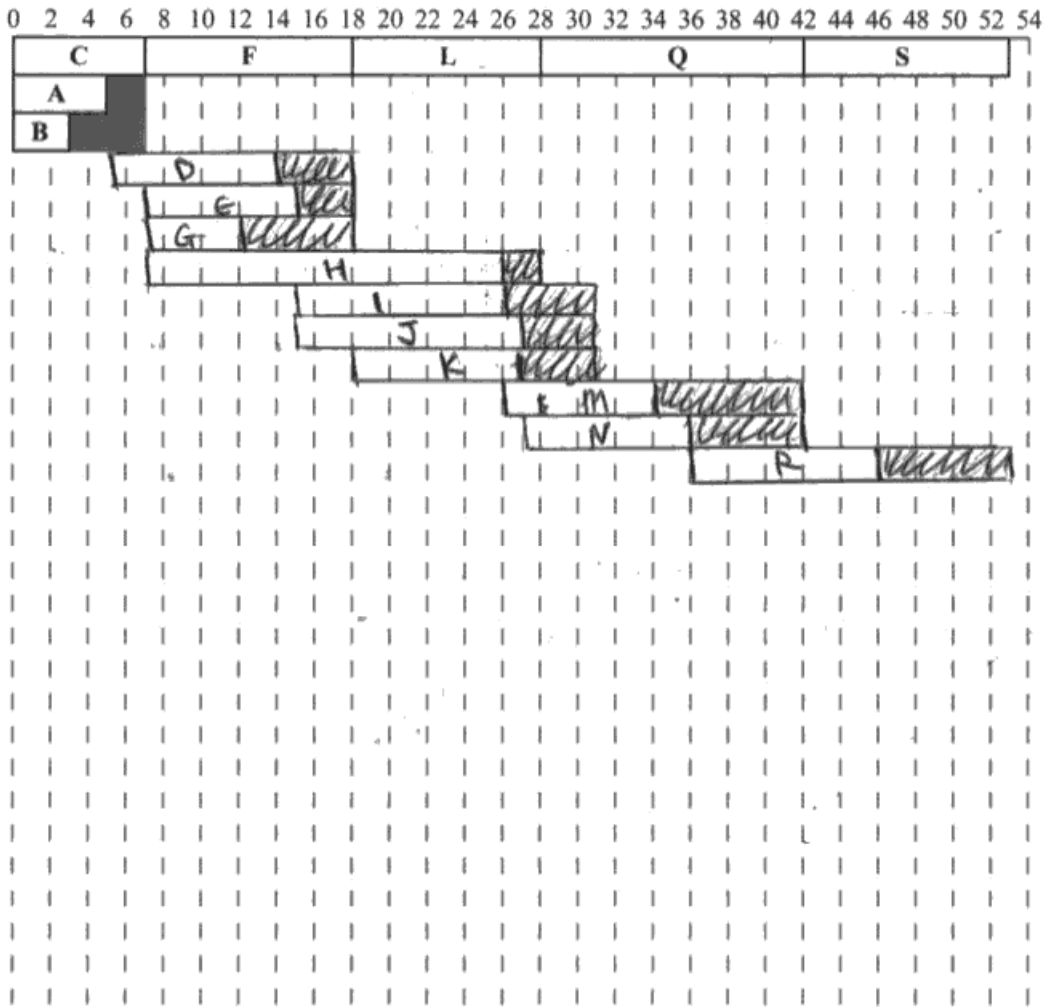


Diagram 2

- (f) 5 workers.  
At ~~time~~ day 25, activities L, H, I, J and K must all be happening.
- (g) Lower bound found in (f) is better because it takes into account when activities overlap.

(Total 17 marks)

Q7

12

TOTAL FOR PAPER: 75 MARKS

END



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