2 Coordinate geometry

1. Find the gradient of the straight line L with equation $\frac{x+2}{3} = \frac{y-2}{4}$

The line M passing through the point (1, –2) is perpendicular to the line L.
Write down the coordinates of the points where line M crosses the axes. (8)

2. (a) Find the gradient of the line joining the points $A(2, 3)$, $B(4, -7)$. (1)

(b) What is the equation of the line AB? (2)

(c) The line AB crosses the x and y axes at points P and Q respectively.
Determine the area of the triangle OPQ where O is the origin. (5)

3. (a) Show that the lines $y = 2x + 3$ and $2y + x = 2$ are perpendicular to each other. (3)

(b) Given that the lines intersect at the point A find the coordinates of this point. (4)

(c) A third line with equation $y = mx + c$ is parallel to the line $y = 2x + 3$ and passes through the point (3, –2).
Determine the value of m and c and hence the equation of the line. (3)

4. (a) Find the equation of the line joining the points $A(0, 4)$ and $B(2, 0)$. (2)

(b) Determine the length of the line AB giving the answer in simplified surd form. (2)

(c) Find the area of the triangle AOB where point O is the origin. (2)

(d) From the point $B(2, 0)$ a line is drawn parallel to the y-axis to a point $C(2, a)$.
The trapezium $OACB$ has an area of 7 units^2.
Determine the value of a. (2)

5. (a) Determine which of these two lines are parallel to each other

$$y + 2x - 4 = 0$$
$$2(y - 2x + 2) = 1$$
$$3(y + 2x) + 1 = 0$$

(2)

(b) The line $y = x$ intersects with the parallel lines at the points A and B.
Find the coordinates of points A and B. (4)

(c) The line $y = x$ also intersects with the non-parallel line at the point C.
Find the coordinates of the point C. (2)

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2 Coordinate geometry

Question Number	Scheme	Marks
1.	$y = \frac{4}{3}x + \frac{14}{3}$	B2
	Gradient of L is $\frac{4}{3}$	A1
	Gradient of M is $-\frac{3}{4}$	A1
	Equation of M is $y + 2 = -\frac{3}{4}(x - 1)$	M1oe
	$4y + 8 = -3x + 3$ $4y + 3x + 5 = 0$	B1
	$\left(0, -\frac{5}{4}\right)$ and $\left(-\frac{5}{3}, 0\right)$	B2ft **8**
2. (a)	Gradient $\frac{-7-3}{4-2} = -5$	A1
(b)	Equation of AB is $y - 3 = -5(x - 2)$ $y = -5x + 13$ or $y + 5x = 13$	M1oe B1
(c)	$P\left(\frac{13}{5}, 0\right)$, $Q(0, 13)$	B2
	Area of the triangle OPQ	
	$= \frac{1}{2} \times \frac{13}{5} \times 13 = \frac{169}{10}$	M1 B1
	$\therefore$ area $= 16.9$	A1 **8**
3. (a)	Gradient of $y = 2x + 3$ is 2 Gradient of $2y + x = 2$ is -0.5 Since $2 \times -0.5 = -1$ Therefore the two lines are perpendicular.	B1 B1 M1 (3)
(b)	$2(2x + 3) + x = 2$ $4x + 6 + x = 2$ $5x = -4$	B1 B1
	$\therefore x = -\frac{4}{5}$ and $y = \frac{7}{5}$, coordinates $\left(-\frac{4}{5}, \frac{7}{5}\right)$	A1 A1 (4)
(c)	$m = 2$, then $-2 = 2 \times 3 + c$ $c = -8$, equation of the line is $y = 2x - 8$	A1 A2 (3) **10**

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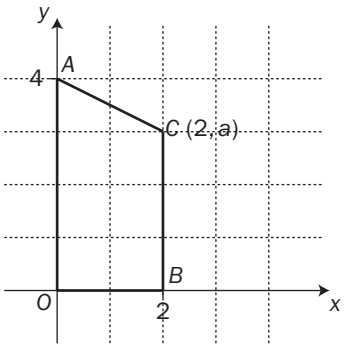
4. (a)	$\frac{0-4}{2-0} = -2$	A1	
	Equation is $y = -2x + 4$	B1ft	(2)
(b)	$AB = \sqrt{(2-0)^2 + (4-0)^2} = \sqrt{20}$	B1	
	$= 2\sqrt{5}$	A1	(2)
(c)	Area of triangle $AOB = \frac{1}{2} \times 2 \times 4 = 4$ units^2	M1 A1	(2)
(d)			
	$\frac{4+a}{2} \times 2 = 7$	M1	
	$a = 3$ units^2	A1	(2)
			8
5. (a)	$y + 2x - 4 = 0$ and $3(y + 2x) + 1 = 0$ (both has gradient -2)	B2	(2)
(b)	$x + 2x - 4 = 0$, then $x = \frac{4}{3}$, therefore $A\left(\frac{4}{3}, \frac{4}{3}\right)$	B1 A1	
	$3(x + 2x) + 1 = 0$, $9x + 1 = 0$, then $x = -\frac{1}{9}$; $B\left(-\frac{1}{9}, -\frac{1}{9}\right)$	B1 A1	(4)
(c)	$2(x - 2x + 2) = 1$, $-2x + 4 = 1$, then $x = \frac{3}{2}$; $C\left(\frac{3}{2}, \frac{3}{2}\right)$	B1 A1	(2)
			8

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3 Quadratic functions

1. (a) Given that $x^2 + 6x + 4 \equiv (x + a)^2 + b$, where a and b are constants, find the value of a and the value of b. (2)

(b) Show that the roots of $x^2 + 6x + 4 = 0$ can be written in the form $p \pm q\sqrt{5}$ and determine the value of p and the value of q. (4)

2. (a) Solve the quadratic equation $4x^2 - 2x - 1 = 0$, writing your answers in a simplified surd form. (2)

(b) Show, by completing the square, that the quadratic can be written in the form

$$4x^2 - 2x - 1 = 4[(x + a)^2 + b]$$

and hence determine the values of a and b. (4)

3. Sketch the curve given by the equation $y = x(x - 4)$ for $-1 \leqslant x \leqslant 5$

(a) Write down the coordinates of the points where the curve intersects the x-axis (4)

(b) Find the coordinates of the turning point indicating the nature of the turning point. (2)

4. Sketch the graph of $y = x^2$ and the graph of $y = 8 - 2x$ on the same axes taking values of x from -5 to $+3$.
Show how you use your graph to solve the equation $x^2 + 2x - 8 = 0$ and hence determine the solutions. (7)

5. (a) The equation $4x^2 + ax + 9 = 0$ has equal roots; find the value of a given that $a < 0$. (3)

(b) The equation $9x^2 + 3kx + k = 0$ has equal roots where $k \neq 0$; find the value of k. (4)

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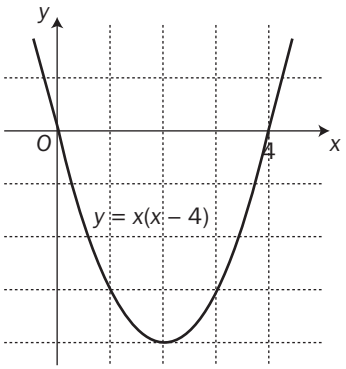
3 Quadratic functions

Question Number	Scheme	Marks
1. (a)	Completing the square of the L.H.S. or expand the bracket of the R.H.S. and compare the coefficients of the x^2 and x. $(x+3)^2 - 3^2 + 4 = (x+3)^2 - 5 \Rightarrow a = 3, b = -5$	A2 (2)
(b)	$(x+3)^2 - 5 = 0$ $(x+3)^2 = 5$ $x + 3 = \pm\sqrt{5}$ $x = -3 \pm \sqrt{5} \Rightarrow$ $p = -3$ and $q = 1$	B1 B1 A2 (4) **6**
2. (a)	Using the quadratic formula: $x = \dfrac{2 \pm \sqrt{(-2)^2 - 4 \times 4 \times (-1)}}{2 \times 4} = \dfrac{2 \pm \sqrt{20}}{8} = \dfrac{1 \pm \sqrt{5}}{4}$	B1 A1 (2)
(b)	$4\left(x^2 - \frac{1}{2}x - \frac{1}{4}\right) = 0$ $\left(x - \frac{1}{4}\right)^2 - \frac{1}{16} - \frac{1}{4} = 0$ $\left(x - \frac{1}{4}\right)^2 - \frac{5}{16} = 0, a = -\frac{1}{4}, b = -\frac{5}{16}$	B1 B1 A2 (4) **6**
3. (a)		B2 (2)
(b)	(0, 0), (4, 0) (2, –4) minimum	B2 (2) B2 (2) **6**

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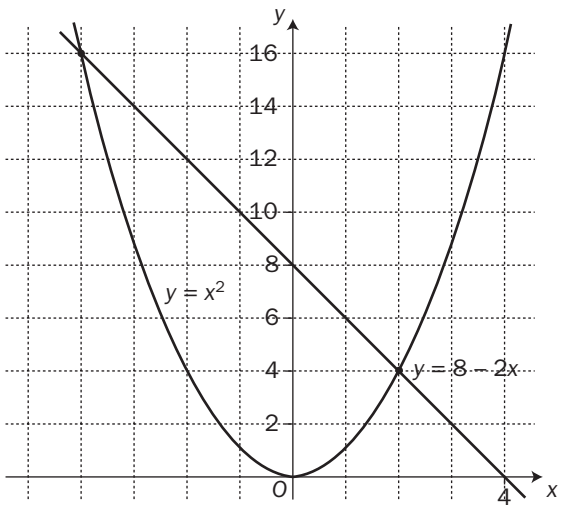
4.	 Subtract from both sides of the equation $x^2 + 2x - 8 = 0$ $2x - 8$ then $x^2 = 8 - 2x$. Solutions are $x = 2$, $x = -4$	B4 B1 A2	**7**
5. (a)	Equal roots then the discriminant = 0. $a^2 - 4 \times 4 \times 9 = 0$ $a^2 = 16 \times 9$ $a < 0 \therefore a = -12$	M1 B1 A1	(3)
(b)	$(3k)^2 - 4 \times 9 \times k = 0$ $9k^2 - 36k = 0$ $9k(k - 4) = 0$ $k \neq 0 \therefore k = 4$	B1 B1 M1 A1	(4) **7**

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4 Quadratic and linear relationships

1. (a) Find, using the substitution method, the values of x and y which satisfy the simultaneous equations

$3x + 2y = -5$ and $x + 4y = 5$ (3)

(b) Solve the simultaneous equations

$2x^2 + 5x + 3(1 + y) = 0$ and $y - 3 = 2x$ (5)

2. Plot the graph of the equation $y = (x - 3)(x + 2)$ taking suitable values of x.
Find the coordinates where the curve crosses the x-axis and the exact coordinates of the turning point stating the nature of the turning point.
The line $y - x + 3 = 0$ intersects with the curve at two points A and B.
Find the coordinates of these two points and identify them clearly on your graph. (9)

3. (a) If $2(3x + 1) - 4(2x - 3) > 0$ find the values of x which satisfy this inequality. (2)

(b) Draw the graph of the inequality represented by $3y < 2x - 1$ and shade the required region on the graph. (3)

4. Find the values of x which satisfy the inequality

$(x + 2)(x - 4) \leqslant 0$

Sketch the graph of $y = (x + 2)(x - 4)$, taking suitable values of x, and shade the region represented by the inequality

$(x + 2)(x - 4) \leqslant y$ (5)

5. Solve the simultaneous equations $x^2 + y^2 = 17$
and $y + 1 = 3x$
using a substitution method. (6)

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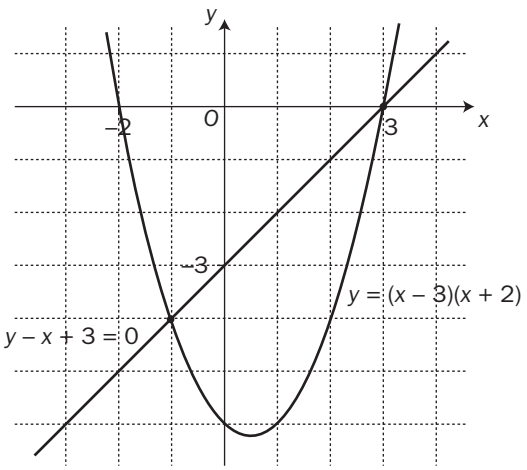
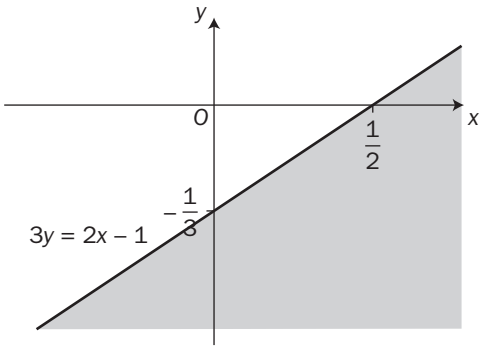
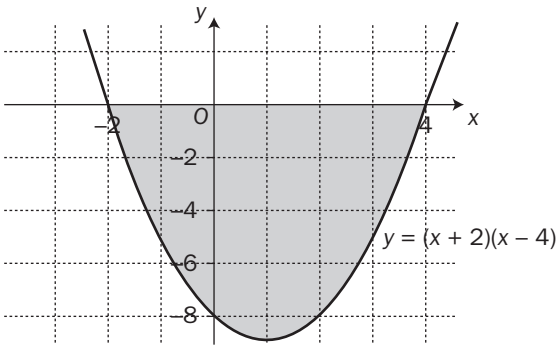
4 Quadratic and linear relationships

Question Number	Scheme	Marks
1. (a)	$x = 5 - 4y$, substitute $x = 5 - 4y$ in $3x + 2y = -5$ $15 - 12y + 2y = -5$ $10y = 20$ $y = 2$ and $x = -3$	 B1oe A1 A1ft (3)
(b)	$y = 2x + 3$ $2x^2 + 5x + 3(1 + 2x + 3) = 0$ $2x^2 + 11x + 12 = 0$ $(2x + 3)(x + 4) = 0$ $x = -\frac{3}{2} \Rightarrow y = 0$ $x = -4 \Rightarrow y = -5$	B1 B1 M1 A1 A1ft A1 A1ft (5) **8**
2.	 (–2, 0), (3, 0), (0.5, –6.25) minimum (–1, –4), (3, 0)	B2 B1 A4 A2 **9**
3. (a)	$6x + 2 - 8x + 12 > 0$ $14 > 2x$ $x < 7$	B1 A1 (2)
(b)		B2 B1 (3) **5**

4.	 $-2 \leqslant x \leqslant 4$	B3 A2 **5**
5.	$y = 3x - 1$ $x^2 + (3x - 1)^2 = 17$ $x^2 + 9x^2 - 6x + 1 = 17$ $10x^2 - 6x - 16 = 0$ $5x^2 - 3x - 8 = 0$ $(5x - 8)(x + 1) = 0$ $x = \frac{8}{5} \Rightarrow y = \frac{19}{5}$ $x = -1 \Rightarrow y = -1$	 B1 M1 A2 A2 **6**

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5 Graphs of functions

1. Sketch the graph of the equation $y = (x - 2)(2x + 1)(x + 3)$, taking suitable values of x. (2)

Find the coordinates where the curve

(a) intersects or touches the x-axis (3)

(b) intersects the y-axis (1)

2. Sketch the graph of the curve

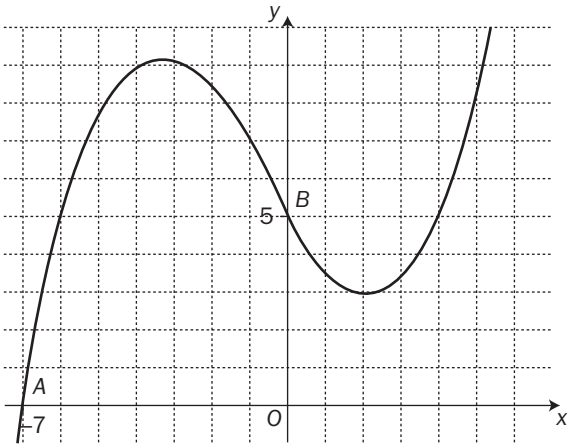
$$y = 2 + \frac{2}{x}$$

Show clearly on the graph the position of the asymptotes.
On the same axes sketch the graph of the curve

$$y = 2 - \frac{2}{x}$$

distinguishing between the two curves.
Describe how the first curve is transformed into the second curve. (5)

3. Make a copy of the graph shown in the diagram where $y = \mathrm{f}(x)$.



The curve crosses the x-axis at the point $A(-7, 0)$ and the y-axis at the point $B(0, 5)$.
On the same axes sketch the graph of

$$y = \mathrm{f}(x - 3)$$

indicating the new position of the points A and B. (3)

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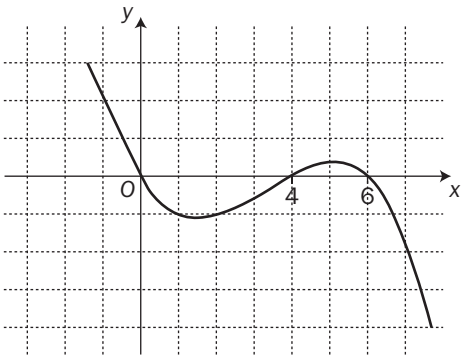
4. The diagram shows the graph of $y = \mathrm{f}(x)$.



On separate axes sketch the graphs of

(a) $y = -\mathrm{f}(x)$ (3)

(b) $y = \mathrm{f}(-x)$ (3)

5. Sketch the graph of the curve given by the equation $y = x^2 - 3x$ and, on the same axes, the graph of the equation $y = x(x^2 - x - 6)$ taking suitable values of x.
Find the coordinates of the points of intersection of the two curves. (6)

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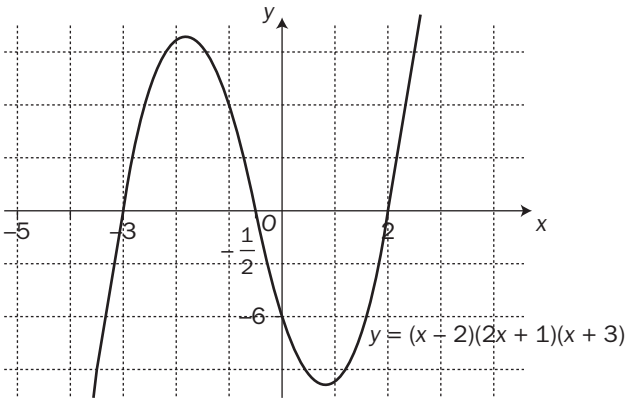
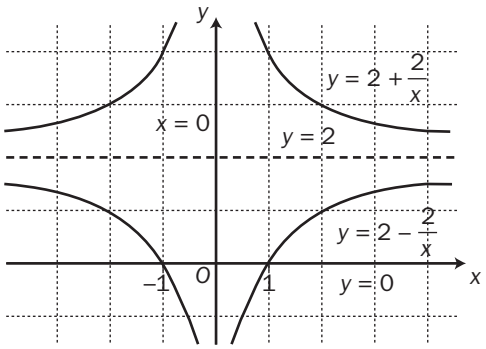
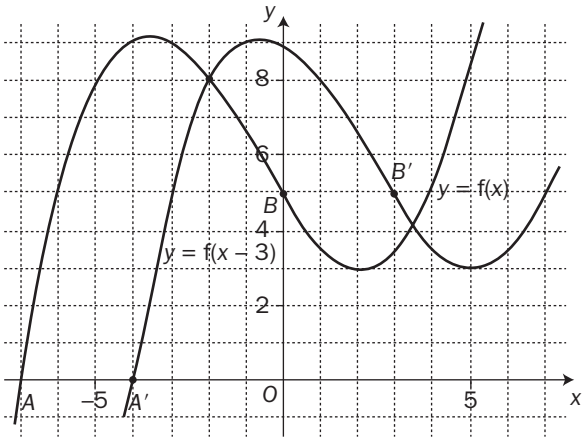
5 Graphs of functions

Question Number	Scheme	Marks
1.		B2
(a)	$(-3, 0)$, $\left(-\frac{1}{2}, 0\right)$ and $(2, 0)$	B3
(b)	$(0, -6)$	B1 **6**
2.		B1 B1
	Asymptotes: $x = 0$ and $y = 2$. Reflection in the y-axis or in the line $y = 2$.	A2 B1 **5**
3.		B3
	New position of the points $A(-4, 0)$ and $B(3, 5)$	**3**

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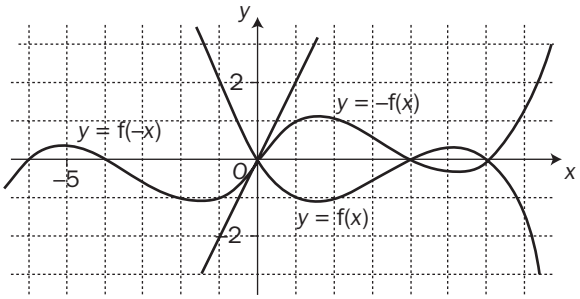
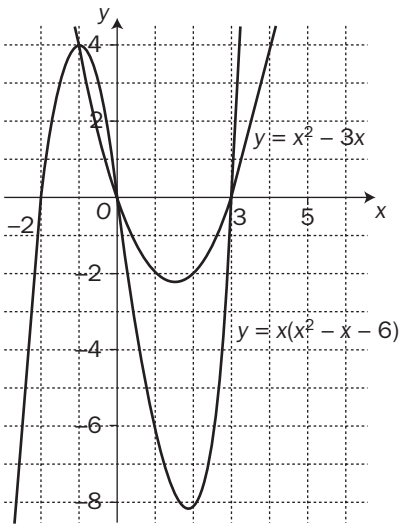
4. (a) (b)	 	B3 B3 **6**
5.	 (–1, 4), (0, 0) and (3, 0).	B1 B2 B3 **6**

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6 Sequences and series

1. (a) The terms in a sequence are generated using the general term

$$u_n = 2n - 3$$

Find the first four terms in this sequence. (2)

(b) The general term of a sequence is $u_n = an^2 + bn + c$ and the first three terms are 3, 6 and 13.
Find the value of the constants a, b and c. (7)

2. (a) A sequence is generated using the recurrence formula

$$u_{n+2} = 2(u_{n+1} + u_n)$$

Given that $u_1 = 2$ and $u_2 = 5$ write down the next three terms in the sequence. (3)

(b) If $u_{n+1} = (k + 1)u_n + 2$, and $u_1 = 3$, $u_2 = 8$, find the value of the constant k. (3)

3. (a) For the arithmetic series 9, 13, 17, 21, …….
write down the general term of the series. (2)

(b) Using the general term deduce the 21st term in the arithmetic series. (2)

(c) Determine the first three consecutive terms in the series which are greater than 500. (4)

4. The general term of an arithmetic series is given by

$$u_n = a + (n - 1)d$$

where a is the first term, d the common difference and n the term.
Find the first four terms in the series when $a = 5$ and $d = -2$.
Determine the sum of the first 20 terms in this series. (6)

5. The rth term of an arithmetic series is $(4r + 1)$

(a) Write down the first three terms in the series. (2)

(b) If d is the common difference of the series write down the value of d. (1)

(c) Show that $\sum_{r=1}^{r=n} (4r + 1) = n(2n + 3)$
and hence find the sum of the first 20 terms in the series. (6)

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6 Sequences and series

Question Number	Scheme	Marks
1. (a)	–1, 1, 3, 5	B2 (2)
(b)	$a + b + c = 3$ ……. (*) $4a + 2b + c = 6$ ….. (**) $9a + 3b + c = 13$ ….. (***) Subtract (*) from (**): $3a + b = 3$ Substitute $3a + b = 3$ in (***): $9 + c = 13$, then $c = 4$. Substitute $c = 4$ in (**): $2a + b = 1$ Subtract $2a + b = 1$ from $3a + b = 3$, then $a = 2$ Substitute $a = 2$, $c = 4$ in (*), $b = -3$	 B2 B1 B1 B1 B1 B1 (7) **9**
2. (a)	$u_3 = 2(5 + 2) = 14$ $u_4 = 2(14 + 5) = 38$ $u_5 = 2(38 + 14) = 104$	B1 B1 B1 (3)
(b)	$u_2 = (k + 1)u_1 + 2$ $8 = 3k + 3 + 2$ $k = 1$	M1 B1 B1 (3) **6**
3. (a)	$u_n = 4n + 5$	B1 B1 (2)
(b)	$u_{21} = 4 \times 21 + 5 = 89$	B1 A1 (2)
(c)	$4n + 5 > 500$, then $n > \frac{495}{4}$ $n = 124$, $u_{124} = 501, 505, 509$	M1oe B1 A2 (4) **8**
4.	$u_n = 5 - 2(n - 1)$, $u_1 = 5$, $u_2 = 3$, $u_3 = 1$ $S_{20} = \frac{(2 \times 5 + 19 \times -2) \times 20}{2} = -28 \times 10 = -280$	B3 M1oe B2 **6**
5. (a)	5, 9, 13	B2 (2)
(b)	$d = 4$	B1 (1)
(c)	First term $a = 5$, $d = 4$, number of terms $= n$, using: $S_n = \frac{(2a + (n - 1)d)n}{2}$ $S_n = \frac{(2 \times 5 + (n - 1) \times 4)n}{2} = \frac{(10 + 4n - 4)n}{2}$ $S_n = (3 + 2n)n$ $S_{20} = (3 + 2 \times 20) \times 20 = 860$	 M1oe B2 B1 B1 A1 (6) **9**

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7 Differentiation

1. A curve is given by the equation $y = ax^2 + bx + 3$ and passes through the point $A(1, 2)$.
The gradient of the curve at this point A is 1.
Find the value of a and the value of b.
Using the values of a and b find the gradient of the curve at the point $B(-1, 8)$. (8)

2. (a) Differentiate $y = \dfrac{4x^2 + 1}{x}$ with respect to x. (3)

(b) Find the values of x at the point where $\dfrac{dy}{dx} = 0$ (3)

(c) Determine the values of $\dfrac{d^2y}{dx^2}$ when $\dfrac{dy}{dx} = 0$ (3)

3. (a) Find the gradient of the tangent drawn to the curve $y = x^3 - x^2$ at the point A where $x = -1$.
Hence find the equation of the tangent at this point A. (6)

(b) Find also the gradient of the normal at the point A and hence its equation. (3)

4. Find the values of the constants a, b and c in the equation $y = ax^2 + bx + c$ given that the curve passes through the point $(0, -4)$, the gradient of the curve is -2 at the point where $x = -\frac{1}{2}$ and the value of $\dfrac{d^2y}{dx^2} = 10$ (7)

5. (a) Given $f(x) = x^3 + \frac{5}{2}x^2 - 2x + 1$, find $f'(x)$. (3)

(b) Find the values of x when $f'(x) = 0$. (3)

(c) Determine the value of $f'(x)$ when $x = -1$ and hence find the equation of the tangent to the curve at this point. (4)

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7 Differentiation

Question Number	Scheme	Marks
1.	Substitute (1, 2) in $y = ax^2 + bx + 3$; $2 = a + b + 3$	B1
	$a + b = -1$ …. (*)	B1
	$\frac{dy}{dx} = 2ax + b$	M1 M1
	$\frac{dy}{dx}$ at A is $2a + b = 1$ ….. (**)	B1
	Subtract (*) from (**): $a = 2$, $b = -3$	B2
	$\frac{dy}{dx} = 4x - 3$, then gradient at B is -7	B1
		8
2. (a)	$y = 4x + x^{-1}$	B1
	$\frac{dy}{dx} = 4 - x^{-2}$	M1 M1 (3)
(b)	$4 - x^{-2} = 0$	B1
	$4x^2 - 1 = 0$	B1
	$x = \pm\frac{1}{2}$	A1 (3)
(c)	$\frac{d^2y}{dx^2} = 2x^{-3}$	M1
	$x = \pm\frac{1}{2} \Rightarrow \frac{d^2y}{dx^2} = \pm 16$	A2 (3)
		9
3. (a)	$\frac{dy}{dx} = 3x^2 - 2x$	M2
	Gradient at $x = -1$ is 5	B1
	If $x = -1$ then $y = -2$, $A(-1, -2)$	B1
	Equation of the tangent at A is $y + 2 = 5(x + 1)$	B1oe
	$y = 5x + 3$	A1 (6)
(b)	gradient of the normal at A is $-\frac{1}{5}$	B1
	equation of the normal at A is $y + 2 = -\frac{1}{5}(x + 1)$	B1oe
	$5y + x + 11 = 0$	A1 (3)
		9

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4.	Substitute (0, –4) in $y = ax^2 + bx + c$, then $c = -4$	A1	
	$\frac{dy}{dx} = 2ax + b$	M1 M1	
	$\frac{dy}{dx}$ at $x = -\frac{1}{2}$ is $-a + b = -2$ …… (*)	B1	
	$\frac{d^2y}{dx^2} = 2a$	B1	
	Therefore $2a = 10$, then $a = 5$, $b = 3$	A1 A1	
			7
5. (a)	$f'(x) = 3x^2 + 5x - 2$	M3	(3)
(b)	$3x^2 + 5x - 2 = 0$		
	$(3x - 1)(x + 2) = 0$	M1oe	
	$x = \frac{1}{3}, -2$	A2	(3)
(c)	$f'(-1) = 3(-1)^2 + 5(-1) - 2 = -4$	B1	
	If $x = -1$ then $y = \frac{9}{2}$	B1	
	Equation of the tangent at $\left(-1, \frac{9}{2}\right)$ is $y - \frac{9}{2} = -4(x + 1)$	B1oe	
	$2y + 8x - 1 = 0$	A1	(4)
			10

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8 Integration

1. Given that $f'(x) = 3x^2 + 2x + 1$, find $f(x)$.
When $y = f(x)$ the values $x = 2$ and $y = 18$ satisfy the equation.
Use this information to write down the equation for y in terms of x.
Hence deduce the value of y when $x = 3$. (6)

2. (a) Find $\int (x + 1)(x^2 - 1)\mathrm{d}x$ (5)

(b) Find $\int \frac{x+1}{x^3}\mathrm{d}x$ (4)

3. The gradient of a curve is given by $f'(x) = 6x^2 - 4x$ and the curve passes through the point $(-1, 6)$.
Find the equation of the curve.
If the curve passes through the point $(3, y)$ find the value of y. (5)

4. (a) Find $\int \sqrt{t}\,\mathrm{d}t$ (2)

(b) Find $\int t\sqrt{t}\,\mathrm{d}t$ (2)

Hence, or otherwise,

(c) find $\int (1 + t)\sqrt{t}\,\mathrm{d}t$ (2)

5. The gradient of a curve is given by $\frac{\mathrm{d}y}{\mathrm{d}x} = ax$

Find the equation of the curve in terms of the constant a and the constant of integration c.

The curve passes through the points $\left(1, \frac{1}{2}\right)$ and $(-2, 8)$.

Determine the values of the constants and hence write down the equation of the curve. (9)

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8 Integration

Question Number	Scheme	Marks
1.	$f(x) = x^3 + x^2 + x + c$ $18 = 8 + 4 + 2 + c$, therefore $c = 4$ $y = x^3 + x^2 + x + 4$ $y = 27 + 9 + 3 + 4 = 43$	M3 B1 B1 B1 **6**
2. (a)	$\int (x^3 + x^2 - x - 1)\,dx = \frac{x^4}{4} + \frac{x^3}{3} - \frac{x^2}{2} - x + c$	B1 M4 (5)
(b)	$\int (x^{-2} + x^{-3})\,dx = -x^{-1} - \frac{x^{-2}}{2} + c$ $\left(\text{Or } \frac{-1}{x} - \frac{1}{2x^2} + c\right)$	B2 M2 (4) **9**
3.	$f(x) = 2x^3 - 2x^2 + c$ $f(-1) = -2 - 2 + c = 6 \Rightarrow c = 10$ $f(x) = 2x^3 - 2x^2 + 10$ $y = f(3) = 54 - 18 + 10 = 46$	M2 A1 B1 A1 **5**
4. (a)	$\int t^{\frac{1}{2}}dt = \frac{2}{3}t^{\frac{3}{2}} + c, \left(or \frac{2t\sqrt{t}}{3} + c\right)$	M1 A1 (2)
(b)	$\int t^{\frac{3}{2}}dt = \frac{2}{5}t^{\frac{5}{2}} + c, \left(or \frac{2t^2\sqrt{t}}{5} + c\right)$	M1 A1 (2)
(c)	$\int \sqrt{t}\,dt + \int t\sqrt{t}\,dt = \frac{2t\sqrt{t}}{3} + \frac{2t^2\sqrt{t}}{5} + c$	A2 (2) **6**
5.	$y = \int ax\,dx = \frac{ax^2}{2} + c$ Substitute $\left(1, \frac{1}{2}\right)$ and $(-2, 8)$ in $y = \frac{ax^2}{2} + c$, to find a and c: $\frac{1}{2} = \frac{a}{2} + c$, then $1 = a + 2c$ or $a = 1 - 2c$ …. (*) $8 = 2a + c$ ….. (**) Substitute (*) in (**): $2 - 4c + c = 8$, then $c = -2$ and $a = 5$ $y = \frac{5x^2}{2} - 2$	M1 A1 M2oe M2oe B2ft A1 **9**

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9 Algebra and functions

1. (a) If $f(x) = 2x^3 + 5x^2 - 21x - 36$ find $f(x) \div (x + 4)$ (2)

(b) Factorise $f(x)$ completely. (2)

(c) Find the values of x which satisfy $f(x) = 0$. (2)

2. (a) If $f(x) = 4x^3 + 16x^2 + 9x - 9$ find the value of $f(-3)$. (2)

(b) Factorise completely the function $f(x) = 4x^3 + 16x^2 + 9x - 9$ (4)

(c) Solve the equation $4x^3 + 16x^2 + 9x - 9 = 0$ (2)

3. (a) Given $f(x) = 4x^3 + 17x^2 + ax - 18$ and $f(-1) = -14$, find the value of the constant a. (2)

(b) Using this value of a show that $f(-2) = 0$. (2)

(c) Factorise the function completely and solve the equation $f(x) = 0$. (4)

4. (a) Use the remainder theorem to find the remainder when the expression $3x^3 - 19x^2 + 38x - 24$ is divided by $(x + 2)$. (2)

(b) Use a suitable method to show that $(x - 3)$ is a factor of the expression. (2)

(c) Solve the equation $3x^3 - 19x^2 + 38x - 24 = 0$. (4)

5. (a) If $f(x) = 4x^3 + ax^2 + bx - 15$ and $f(2) = 63$, $f(-1) = -6$, find the values of the constants a and b. (6)

(b) Using the values of a and b factorise the expression completely. (5)

(c) The curve given by the equation $y = 4x^3 + ax^2 + bx - 15$ crosses the x-axis in three points. Find the values of x where this occurs. (2)

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9 Algebra and functions

Question Number	Scheme	Marks	
1. (a)	By 'inspection' or algebraic division: $f(x) \div (x + 4) = (2x^2 - 3x - 9)$	B2	(2)
(b)	$f(x) = (x + 4)(2x + 3)(x - 3)$	A2	(2)
(c)	$x = -4,\ x = -\frac{3}{2}$ or $x = 3$	A2	(2)
			6
2. (a)	$f(-3) = 4 \times (-27) + 16 \times 9 + 9 \times (-3) - 9 = 0$	A2	(2)
(b)	Since $f(-3) = 0$, therefore $(x + 3)$ is a factor of $f(x)$: By 'inspection' or algebraic division: $f(x) = (x + 3)(4x^2 + 4x - 3)$ $f(x) = (x + 3)(2x - 1)(2x + 3)$	B1 M2oe B1	
(c)	$f(x) = 0$, then $x = -3$, $x = 0.5$, $x = -1.5$	A2	(6)
			8
3. (a)	$f(-1) = -4 + 17 - a - 18 = -14$ $a = 9$	M1 A1	 (2)
(b)	$f(x) = 4x^3 + 17x^2 + 9x - 18$ $f(-2) = -32 + 68 - 18 - 18 = 0$	B1 A1	 (2)
(c)	Since $f(-2) = 0$, therefore $(x + 2)$ is a factor of $f(x)$: By 'inspection' or algebraic division: $f(x) = (x + 2)(4x^2 + 9x - 9)$ $f(x) = (x + 2)(4x - 3)(x + 3)$ $f(x) = 0$, then $x = -2$, $x = \frac{3}{4}$, $x = -3$	 M1 M1 A2	 (4)
			8
4. (a)	Let $f(x) = 3x^3 + 19x^2 + 38x - 24$ $f(-2) = -24 - 76 - 76 - 24 = -200$	M1 A1	 (2)
(b)	$f(3) = 81 - 171 + 114 - 24 = 0$ Since $f(x) = 0$, then $(x - 3)$ is a factor of $f(x)$.	M1oe B1	 (2)
(c)	$f(x) = (x - 3)(3x^2 - 10x + 8)$ $f(x) = (x - 3)(3x - 4)(x - 2)$ $f(x) = 0$, then $x = 3$, $x = \frac{4}{3}$, $x = 2$.	M1 B1 A2	 (4)
			8

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5. (a)	$f(2) = 32 + 4a + 2b - 15 = 63$	M1	
	$4a + 2b = 46$, then $2a + b = 23$ …. (*)	A1	
	$f(-1) = -4 + a - b - 15 = -6$	M1	
	$a - b = 13$ …… (**)	A1	
	Adding (*) and (**):		
	$3a = 36$, then $a = 12$, and $b = -1$	A2	(6)
(b)	$f(x) = 4x^3 + 12x^2 - x - 15$	B1	
	$f(1) = 4 + 12 - 1 - 15 = 0$	M1	
	Therefore $(x - 1)$ is a factor of $f(x)$	B1	
	$f(x) = (x - 1)(4x^2 + 16x + 15)$	M1oe	
	$f(x) = (x - 1)(2x + 5)(2x + 3)$	A1	(5)
(c)	$y = f(x)$ crosses the x-axis at: $x = 1$, $x = -\frac{5}{2}$, $x = -\frac{3}{2}$	A2	(2)
			13

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10 Coordinate geometry

1. (a) Find the equation of the perpendicular bisector of the line joining points $A(-1, 2)$ and $B(3, -4)$. (7)

(b) The perpendicular bisector of the line CD has equation $2y + x = 4$.
If the lines AB and CD are chords of a circle find the coordinates of the centre of the circle. (4)

2. (a) The ends of a diameter of a circle have coordinates $(3, -5)$ and $(9, 7)$.
Find the equation of the circle. (4)

(b) If this circle crosses the y-axis at the points A and B determine the coordinates of the points A and B. (3)

(c) Find the area of the triangle ABC where C is the centre of the circle. (3)

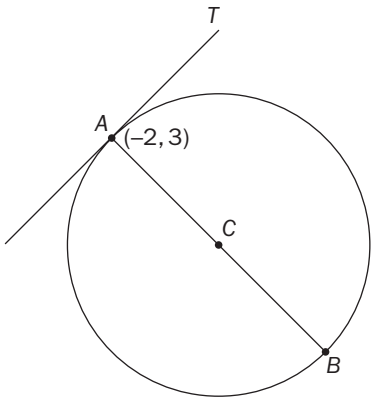
3. The points $A(3, 2)$, $B(7, -2)$ and $C(2a, a)$ lie on the circumference of a circle.
If AB is a diameter of the circle find the coordinates of the centre of the circle and the exact radius of the circle.
Find the least value of a, correct to two decimal places. (6)

4. (a) The equation of the circle is $(x + 3)^2 + (y - 3)^2 = 9$.
Write down the coordinates of the centre of the circle and the radius of the circle. (2)

(b) The line $y = x + 3$ intersects the circle at the points A and B.
Find the coordinates of the points A and B. (4)

(c) The line $y = x + a$ is a tangent to the circle. Show that there are two possible values of a and find the exact values. (6)

5. (a) The tangent AT to the circle, with centre C, has gradient of 1.5.
If the point A has coordinates $(-2, 3)$ find the equation of the radius AC. (2)



(b) The line AC is produced to intersect the circle at the point B.
If the x-coordinate of the centre of the circle is $x = 4$, find the coordinates of the point B. (4)

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10 Coordinate geometry

Question Number	Scheme	Marks
1. (a)	Midpoint of AB is $\left(\frac{-1+3}{2}, \frac{2-4}{2}\right) = (1, -1)$	M1 A1
	Gradient of AB is $\frac{-4-2}{3+1} = -\frac{6}{4} = -\frac{3}{2}$	M1 A1
	Gradient of the perpendicular is $\frac{2}{3}$	B1ft
	Equation: $y + 1 = \frac{2}{3}(x - 1)$	M1oe
	$3y = 2x - 5$	A1 (7)
(b)	The perpendicular bisector of a chord passes through the centre of the circle.	B1
	Therefore, the centre of the circle is the point of intersection of $3y = 2x - 5$ and $2y + x = 4$	
	Substitute $x = 4 - 2y$ in $3y = 2x - 5$	M1oe
	$3y = 8 - 4y - 5$, then $y = \frac{3}{7}$ and $x = \frac{22}{7}$	
	Centre $\left(\frac{22}{7}, \frac{3}{7}\right)$	A2 (4)
		11
2. (a)	Centre of the is the midpoint of $AB = (6, 1)$	A1
	Radius of the circle $= \sqrt{(9-6)^2 + (7-1)^2} = \sqrt{45}$	A1
	Equation of the circle is $(x - 6)^2 + (y - 1)^2 = 45$	A2 (4)
(b)	If $x = 0$, then $36 + (y - 1)^2 = 45$, $(y - 1)^2 = 9$, $(y - 1) = \pm 3$	M1
	$y = 4$, $y = -2$, $A(0, 4)$ and $B(0, 2)$	A2 (3)
(c)	$AC = BC$ (radii of the circle)	B1
	ACB is an isosceles triangle, base AB and height = 6	B1
	Area of the triangle $= \frac{1}{2} \times 6 \times 6 = 18$ units^2	A1 (3)
		10
3.	Centre of the circle is the midpoint of $AB = (5, 0)$	A1
	Radius $= \sqrt{4+4} = 2\sqrt{2}$	M1 A1
	$(2a - 5)^2 + a^2 = 8$	M1
	$5a^2 - 20a + 17 = 0$	B1
	$a = 2.77$ (2 dp.) or $a = 1.23$ (2 dp.)	A2
		6

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4. (a)	Centre (–3, 3), radius 3	A2	(2)
(b)	Substitute $y = x + 3$ in the equation of the circle:		
	$(x + 3)^2 + x^2 = 9$	B1	
	$2x^2 + 6x = 0$	B1	
	$x(x + 3) = 0$	B1	
	$x = -3$, then $y = 0$; $A(-3, 0)$	A1ft	
	$x = 0$, then $y = 3$; $B(0, 3)$	A1ft	(4)
(c)	$(x + 3)^2 + (x + a - 3)^2 = 9$	M1	
	$x^2 + 6x + 9 + x^2 + 2(a - 3) + (a - 3)^2 = 9$		
	$2x^2 + 2ax + (a - 3)^2 = 0$	B1oe	
	Since the line touches the circle, therefore the discriminant = 0		
	$(2a)^2 - 4 \times 2 \times (a - 3)^2 = 0$	M1	
	$a^2 - 12a + 18 = 0$		
	$(a - 6)^2 = 18$, then $a = 6 \pm 3\sqrt{2}$	M1oe A2	(6)
			12
5. (a)	Gradient of AC is $-\frac{2}{3}$	A1	
	$y - 3 = -\frac{2}{3}(x + 2)$; $3y + 2x = 5$	A1	(2)
(b)	Since C is the midpoint of AB, then	B1	
	$\frac{-2 + x}{2} = 4$	M1oe	
	Therefore the x-coordinate of B is 10.	A1	
	Substitute $x = 10$ in $3y + 2x = 5$:		
	$y = 5$	A1	(4)
			6

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11 Exponentials and logarithms

1. (a) Find the value of (i) 2^4 (ii) 2^{-4} (2)

(b) Sketch the graph of the equation $y = 2^x$ taking values of x from

$x = -4$ to $x = 4$.

Hence, or otherwise, sketch the graph of $y = 2^{-x}$ on the same axes. (4)

2. (a) Write $3^2 = 9$ in a logarithmic form. (1)

(b) Write $\log_3 81 = 4$ in an index form. (1)

(c) Find the base a of the logarithm when $\log_a 125 = 3$. (2)

(d) Using your calculator evaluate $\log_{10} 50$ to three decimal places. (2)

3. (a) Simplify the expression $3 \log 2 + 2 \log 3 - \log 4$ writing the answer in the form $\log N$ where N is an integer. (3)

(b) Simplify the logarithmic expression $2 \log a - \frac{1}{2} \log b + \log c$ giving your answer as a single logarithmic term. (3)

(c) Solve the equation $\log_2(x + 1) - \log_2 x = 3$, giving your answer correct to two decimal places. (4)

4. Solve the equation giving the value(s) of x correct to three significant figures.

(a) $5^{2x} = 2(5)^x$ (4)

(b) $4(7^{2x}) - 4(7^x) + 1 = 0$ (4)

5. Solve the equations

(a) $4^{2x+2} - 2(4^{x+1}) + 1 = 0$ (4)

(b) $3(5^{2x}) - 13(5^x) + 14 = 0$ (6)

writing your answers correct to three significant figures.

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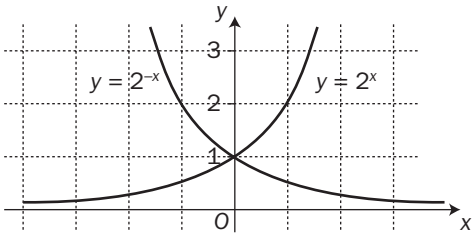
11 Exponentials and logarithms

Question Number	Scheme	Marks
1. (a)	(i) 16	A1
	(ii) 0.0625	A1 (2)
(b)		B2
		B2 (4) **6**
2. (a)	$\log_3 9 = 2$	A1 (1)
(b)	$3^4 = 81$	A1 (1)
(c)	$5^3 = a^3$; $a = 5$	M1 A1 (2)
(d)	1.69897 = 1.699 (3 dp.)	B1 A1 (2) **6**
3. (a)	$\log 8 + \log 9 - \log 4 = \log \frac{8 \times 9}{4} = \log 18$	M2 A1 (3)
(b)	$\log a^2 - \log b^{\frac{1}{2}} + \log c = \log \frac{a^2c}{\sqrt{b}}$	M2 A1 (3)
(c)	$\log_2\left(\frac{x+1}{x}\right) = 3$	M1 M1oe
	$x + 1 = 2^3x$	B1
	$x = \frac{1}{7}$	A1 (4) **10**
4. (a)	$2x \log 5 = \log 2 + x \log 5$	M1
	$x \log 5 = \log 2$	B1
	$x = \frac{\log 2}{\log 5} = 0.431$ (3 dp.)	B1 A1 (4)
(b)	$(2 \times 7^x - 1)^2 = 0$	M2oe
	$x = \frac{-\log 2}{\log 7} = -0.356$ (3 dp.)	B1 A1 (4) **8**

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5. (a)	$16 \times 4^{2x} - 8 \times 4^{x} + 1 = 0$ $(4 \times 4^{x} - 1)^{2} = 0$ $x = \dfrac{-\log 4}{\log 4} = -1$	B1 M1 B1 A1 (4)
(b)	$(3 \times 5^{x} - 7)(5^{x} - 2) = 0$ $x = \dfrac{\log 7 - \log 3}{\log 5} = 0.5269\,(3\text{ dp.})$ $x = \dfrac{\log 2}{\log 5} = 0.431\,(3\text{ dp.})$	M2 B1oe A1 B1 A1 (6) **6**

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12 Trigonometry

1. (a) Find the values of x such that $0° \leqslant x \leqslant 360°$ for

$$\sin(x - 20°) = 0.3$$

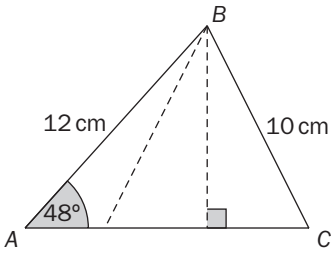
giving your answer correct to one decimal place. (4)

(b) If $\tan^2 x = 0.04$ find the values of x for $0° \leqslant x \leqslant 180°$ which satisfy the equation, giving your answers correct to one decimal place. (3)

2. (a) Write the equation $3\cos^2 x + \sin x = 1$ in terms of $\sin x$ only. (2)

(b) Hence, or otherwise, find the values of x, in the interval 0° to 360°, which satisfy the equation $3\cos^2 x + \sin x = 1$.
Give your answers correct to one decimal place. (6)

3. In triangle ABC, side $AB = 12$ cm, $BC = 10$ cm and angle $BAC = 48°$.



(a) Show that there are two possible sizes for angle BCA and determine their values correct to one decimal place. (5)

(b) Find the two possible lengths of the side AC of the triangle giving your answers correct to one decimal place. (5)

4. Sketch the graph of $y = \cos x$ and, on the same axes, the graph of $y = -\cos x$ for values of x in the interval 0 to 2π radians.
Describe the transformation which has taken place from the first graph to obtain the second graph. (6)

5. The chord AB of a circle centre O subtends an angle of 288° at the centre of the circle.
The length of the major arc AB is 17.8 cm.
Find the radius of the circle.
Find also the area of the major sector of the circle writing your area correct to three significant figures. (5)

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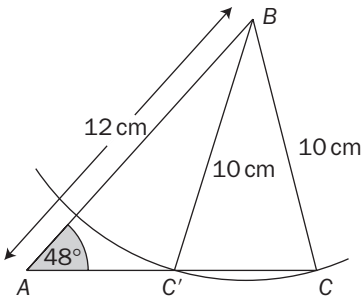
12 Trigonometry

Question Number	Scheme	Marks
1. (a)	$x - 20° = 17.5° \Rightarrow x = 37.5°$ $x - 20° = 162.5° \Rightarrow x = 182.5°$	B1 A1 B1 A1 (4)
(b)	$\tan x = \pm 0.2$ $x = 11.3°;\ x = 168.7°$	B1 A1 A1 (3)
		7
2. (a)	$3 - 3\sin^2 x + \sin x = 1$ $3\sin^2 x - \sin x - 2 = 0$	M1 B1 (2)
(b)	$(3\sin x + 2)(\sin x - 1) = 0$ $\sin x = 1 \Rightarrow x = 90°$ $\sin x = -\frac{2}{3} \Rightarrow x = 221.8°;\ x = 318.2°$	M1 B1 A1 B1 A2 (6)
		8
3. (a)	 Using the sine rule: $\frac{\sin c}{12} = \frac{\sin 48°}{10}$ $\sin c = 6 \times \frac{\sin 48°}{5} \Rightarrow c = 63.1°$ or $c = 116.9°$	B1 M1 B1 A2 (5)
(b)	If $c = 63.1°$, then angle $ABC = 68.9°$, using the sine rule $\frac{AC}{\sin 68.9°} = \frac{10}{\sin 48°}$, $AC = \sin 68.9° \times \frac{10}{\sin 48°} = 12.6\,\text{cm}$ If $c = 116.9°$, then angle $ABC = 15.1°$, using the sine rule $AC = \sin 15.1° \times \frac{10}{\sin 48°} = 3.5\,\text{cm}$	A1 M1 A1 A1 A1 (5)
		10

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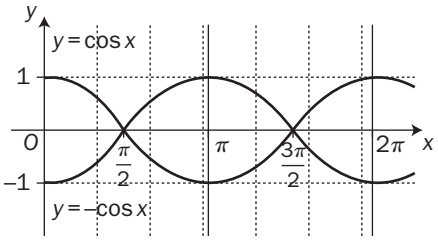
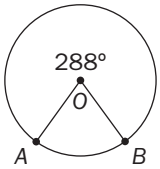
4.	 Reflection in the x-axis	B3 B2 B1 **6**
5.	 arc $= r\theta^c$ $288° \equiv 5.027^c$, $r = \dfrac{17.8}{5.027} = 3.54\,\text{cm}$ Area $= \dfrac{1}{2} \times (3.54)^2 \times 5.027 = 31.5\,\text{cm}^2$	A1 M1 A1 M1 A1ft **5**

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13 Sequences and series

1. (a) The first term in a geometric series is 4 and the common ratio is $\frac{1}{2}$.
Write down the first four terms in the series. (1)

(b) Find the sum of the first ten terms of this geometric series giving your answer correct to four decimal places. (2)

(c) Determine the difference between the sum to infinity and the sum to ten terms of this series giving your answer correct to four decimal places. (3)

2. (a) For the geometric series $5, -\frac{5}{3}, \frac{5}{9}, -\frac{5}{27}$ find the common ratio. (2)

(b) Write down the next two terms in this series. (2)

(c) Find the sum to infinity of this series. (2)

3. (a) Write down all the terms in the expansion of $(2 + x)^4$ giving your answer in the simplest form. (3)

(b) If terms involving x^3 and higher may be neglected show that

$$(1 + 2x)(2 + x)^4 \approx a + bx + cx^2$$

and find the values of the constants a, b and c. (3)

4. (a) In the expansion of $\left(1 - \frac{x}{2}\right)^n$ the coefficient of x^2 is 9, where n is a positive integer.
Find the value of n. (5)

(b) Using this value of n find the coefficient of the term in x^3. (2)

5. (a) In the expansion of $(1 + ax)^8$ the third term is $448x^2$.
Find the value of the constant a. (3)

(b) Taking the first four terms in the expansion of $(1 + ax)^8$ and, using the value of the constant a, find an approximate value of $(1.4)^8$ writing your result to three decimal places. (6)

(c) What is the percentage error in taking this approximate value of $(1.4)^8$ compared with the exact value?
Write the percentage to the nearest integer. (2)

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13 Sequences and series

Question Number	Scheme	Marks	
1. (a)	$4, 2, 1, \frac{1}{2}$	A1	(1)
(b)	$S_{10} = \frac{4(1-(0.5)^{10})}{1-0.5} = 7.9922$ (4 dp.)	M1 A1	(2)
(c)	$S_\infty = \frac{2}{1-0.5} = 8$	M1 A1	
	$S_\infty - S_{10} = 0.0078$	A1	(3)
			6
2. (a)	$d = 5 \div -\frac{5}{3} = -\frac{1}{3}$	M1 A1	(2)
(b)	$\frac{5}{81}, -\frac{5}{243}$	A2	(2)
(c)	$S_\infty = 5 \div \left(1 - \frac{-1}{3}\right) = \frac{15}{4}$ or 3.75	M1 A1	(2)
			6
3. (a)	$16 + 32x + 24x^2 + 8x^3 + x^4$	A3	(3)
(b)	$(1 + 2x)(16 + 32x + 24x^2 + 8x^3 + x^4) \approx 16 + 64x + 88x^2$		
	$a = 16, b = 64, c = 88$	A3	(3)
			6
4. (a)	$\left(1 - \frac{x}{2}\right)^n = 1 - n \times \frac{x}{2} + \frac{n(n-1)}{2} \times \left(-\frac{x}{2}\right)^2 + \ldots$	M1 A1	
	$\therefore \frac{n(n-1)}{8} = 9$	B1	
	$n^2 - n - 72 = 0$	B1	
	$(n - 9)(n + 8) = 0$		
	Since n is a positive, therefore $n = 9$	A1	(5)
(b)	Coefficient of x^3: $\frac{n(n-1)(n-2)}{6} \times \left(-\frac{1}{2}\right)^3 = -\frac{21}{2}$	M1 A1	(2)
			7
5. (a)	$\frac{8 \times 7}{2} a^2 = 448,\ a = \pm 4$	M1 A2	(3)
(b)	$1 + 8ax + 28a^2x^2 + 56a^3x^3 + \ldots$	M3	
	Let $x = 0.1$, then	B1	
	$(1 + 4 \times 0.1)^8 \approx 1 + 32 \times 0.1 + 448 \times 0.01 + 3584 \times 0.001 = 12.264$	M1 A1	(6)
(c)	Percentage error: $\frac{(1.4)^8 - 12.264}{(1.4)^8} \times 100 = 17\%$ (nearest integer)	M1 A1	(2)
			11

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14 Differentiation

1. (a) Find $\frac{dy}{dx}$ when $y = x(3 - x^2)$ (3)

(b) Determine the values of x for which $\frac{dy}{dx} = 0$ (2)

(c) Find the values of x for which y represents an increasing function. (3)

2. (a) If $y = 4x^3 + 9x^2 - 84x$ find the values of x for which $\frac{dy}{dx} = 0$ (6)

(b) Find the coordinates of the turning points on this curve given by the equation

$y = 4x^3 + 9x^2 - 84x.$ (2)

(c) Find the second derivative $\frac{d^2y}{dx^2}$ and use it to determine the nature of the turning points. (6)

3. A closed container in the shape of a cuboid is made from card of negligible thickness.
It is designed such that the length of the container is three times its width.
The container is to have a fixed volume of 562.5 cm^3 whilst the amount of card used is to be a minimum.
Find the dimensions of the container and its surface area. (10)

4. A curve is given by the equation $y = 6x^2 + x - 15$

(a) Find the coordinates of the points where the curve crosses the x and y axes. (4)

(b) Use calculus to show that the curve has a minimum turning point and find the coordinates of the turning point. (5)

(c) Sketch the curve. (2)

5. If $x - y = 9$, find the maximum value of xy^2. (9)

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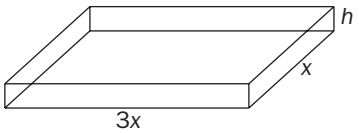
14 Differentiation

Question Number	Scheme	Marks
1. (a)	$y = 3x - x^3$, $\frac{dy}{dx} = 3 - 3x^2$	B1 A2 (3)
(b)	$\frac{dy}{dx} = 0$, then $x = \pm 1$	A2 (2)
(c)	Increasing function, then $\frac{dy}{dx} > 0$, $3 - 3x^2 > 0$	B1
	$x^2 < 1$, therefore $-1 < x < 1$	A2 (3)
		8
2. (a)	$\frac{dy}{dx} = 12x^2 + 18x - 84 = 0$	M3
	$2x^2 + 3x - 14 = 0$	
	$(2x + 7)(x + 2) = 0$, $x = -\frac{7}{2}$, $x = 2$	M1 A2 (6)
(b)	$\left(-\frac{7}{2}, \frac{981}{3}\right)$, $(2, -100)$	A2 (2)
(c)	$\frac{d^2y}{dx^2} = 24x + 18$	M2
	If $x = -\frac{7}{2}$, then $\frac{d^2y}{dx^2} = -66 < 0$, max. at $\left(-\frac{7}{2}, \frac{931}{4}\right)$	B1 A1
	If $x = 2$, then $\frac{d^2y}{dx^2} = 30 > 0$, min. at $(2, -100)$	B1 A1 (6)
		14
3.		
	Volume $= 3x^2h = 562.5$, then $h = \frac{187.5}{x^2}$	M1
	Surface area, $A = 6x^2 + 8x \times \frac{187.5}{x^2} = 6x^2 + \frac{1500}{x}$	M1 A1
	$A = 6x^2 + 1500x^{-1}$, then $\frac{dA}{dx} = 12x - 1500x^{-2} = 0$	M2
	$12x^3 = 1500$, $x = 5$ cm	B1 A1
	$\frac{d^2A}{dx^2} = 12 + 3000x^{-3}$, $\frac{d^2A}{dx^2} > 0$ at $x = 5$, then min. at $x = 5$	B1
	Width = 5 cm and length = 15 cm;	A1
	min. surface area = 450 cm^2	A1
		10

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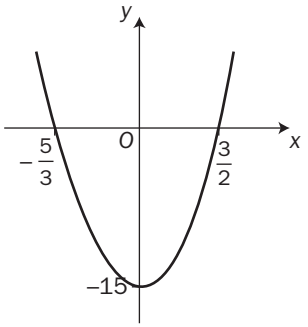
4. (a)	$y = (3x + 5)(2x - 3)$ $(0, -15), \left(-\frac{5}{3}, 0\right), \left(\frac{3}{2}, 0\right)$	B1 A3 (4)
(b)	$\frac{dy}{dx} = 12x + 1$ If $\frac{dy}{dx} = 0$, then $x = -\frac{1}{12}$ $\frac{d^2y}{dx^2} = 12 > 0$, therefore min. at $x = -\frac{1}{12}, \left(-\frac{1}{12}, -\frac{361}{24}\right)$	M2 A1 B1 A1 (5)
(c)		B2 (2) **11**
5.	Let $A = xy^2$, substitute $x = y + 9$ in $A = xy^2$ $A = y^3 + 9y^2$ $\frac{dA}{dy} = 3y^2 + 18y = 0$, then $y = 0$ or $y = -6$ $\frac{d^2A}{dy^2} = 6y + 18$ and $\frac{d^2A}{dy^2} < 0$ at $y = -6$ Therefore A is max. at $y = -6$, $x = 3$ Maximum value of xy^2 is 108	B1 B1oe M2oe M2oe A2 A1 **9**

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15 Integration

1. (a) Find the equation for y when $y = \int (6x^2 + 6x - 1)\,dx$ given that $x = -2$, $y = 2$ satisfy the equation. (4)

(b) Evaluate the integral $\int_1^2 (6x^2 + 6x - 1)\,dx$ (2)

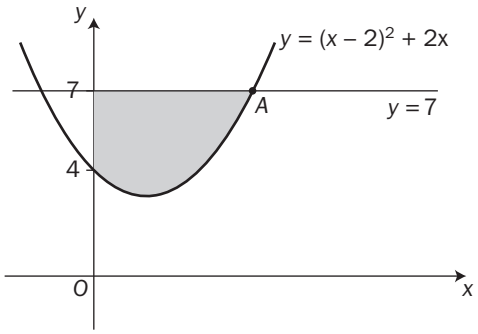
2. (a) Find the x values where the curve given by the equation $y = -2x^2 + 9x - 4$ crosses the x-axis. (3)

(b) Use calculus to show that there is a maximum turning point and find the coordinates of the turning point. (5)

(c) Use the information from parts **a** and **b** to sketch the curve. (2)

(d) Find the area enclosed between the curve and the x-axis, giving your answer correct to two decimal places. (5)

3. The curve given by the equation $y = (x - 2)^2 + 2x$ is shown in the figure.



(a) The line $y = 7$ intersects the curve at point A where the x value is positive. Find the coordinates of the point A. (1)

(b) Find the area of the segment which lies between the curve $y = (x - 2)^2 + 2x$, the straight line $y = 7$ and the y-axis. (6)

4. A turbo charged racing car is accelerating along a track and its velocity v m s^{-1} is recorded every second.
The results are shown in the table below where t is the time in seconds

t	0	1	2	3	4	5	6
v	0	4.5	10	16.5	24	32.5	42

The results are plotted on a graph and the area between the curve, the horizontal axis and the lines representing the values $t = 0$ and $t = 6$, is the distance travelled by the car in 6 seconds.
Use the trapezium rule to estimate the distance travelled by the car in 6 seconds. (4)

5. Use the trapezium rule to evaluate the integral $\int_1^5 \frac{10}{x+1}\,dx$ taking eight strips.
Write your answer correct to two decimal places. (10)

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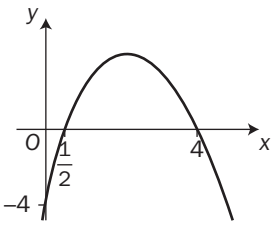
15 Integration

Question Number	Scheme	Marks
1. (a)	$y = 2x^3 + 3x^2 - x + c$ $2 = -16 + 12 + 2 + c \Rightarrow c = 4$ $y = 2x^3 + 3x^2 - x + 4$	A2 A1 B1 (4)
(b)	$(2x^3 + 3x^2 - x]_1^2 = 26 - 4 = 22$	B1 A1 (2) **6**
2. (a)	$2x^2 - 9x + 4 = 0$ $(2x - 1)(x - 4) = 0$ $x = \frac{1}{2}, x = 4$	 M1oe A2 (3)
(b)	$\frac{dy}{dx} = -4x + 9 = 0$ $x = \frac{9}{4}$ $\frac{d^2y}{dx^2} = -4 < 0$, therefore max. point $\left(\frac{9}{4}, \frac{49}{8}\right)$	M2 A1 B1 A1 (5)
(c)		B2 (2)
(d)	Area $= \int_{\frac{1}{2}}^{4} (-2x^2 + 9x - 4)\,dx = \left(\frac{-2x^3}{3} + \frac{9x^2}{2} - 4x\right]_{\frac{1}{2}}^{4}$ $= \frac{40}{3} + \frac{23}{24} = 14.29$ units^2 (2 dp.)	M3 A2 (5) **15**

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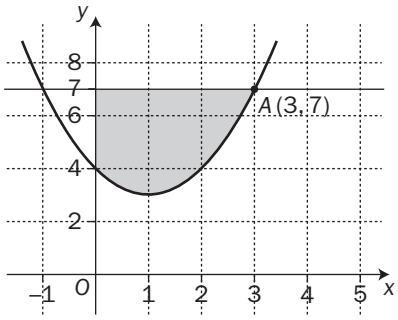
3.



(a) A(3, 7) — A1

(b) $\text{Area} = \int_0^3 (7 - y)\,dx = \int_0^3 (7 - (x-2)^2 - 2x)\,dx = \int_0^3 (-x^2 + 2x + 3)\,dx$ — M2oe

$= \left(-\frac{1}{3}x^3 + x^2 + 3x\right]_0^3 = -9 + 9 + 9 = 9 \text{ units}^2$ — M3 A1 (6)

7

4. $\text{Area} \approx \frac{1}{2}[(0 + 42) + 2(4.5 + 10 + 16.5 + 24 + 32.5)] = 108.5 \text{ m}$ — M3 A1

4

5. $y = \frac{10}{x+1}$ — B1

x	1	1.5	2	2.5	3	3.5	4	4.5	5
y	5	4	$\frac{10}{3}$	$\frac{20}{7}$	2.5	$\frac{20}{9}$	2	$\frac{20}{11}$	$\frac{5}{3}$

B4

$\text{Area} \approx \frac{0.5}{2}\left[\left(5 + \frac{5}{3}\right) + 2\left(4 + \frac{10}{3} + \frac{20}{7} + 2.5 + \frac{20}{9} + 2 + \frac{20}{11}\right)\right]$ — M3

$= 11.03 \text{ units}^2$ (2 dp.) — A2

10

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